RATIONAL NUMBERS

1.1 Definition of Rational Numbers:

Question:

What are rational numbers?

Answer:

A number that can be written as $\frac{p}{q}$, where p and q are integers and q $\neq$ 0, is known as **Rational Number.**

Example: $\frac{2}{3}, \frac{4}{5}, -\frac{9}{11}$, 12, -18 etc.

If the signs of numerator and denominator are either both positive or both negative, the rational number is known as **Positive Rational Number**.

Example: $\frac{-2}{-7}, \frac{14}{25}$ etc.

If the signs of numerator and denominator are opposite to each other, the rational number is known as **Negative Rational Number.**

Example: $\frac{-2}{9}, \frac{4}{-17}$ etc.

1.2 Types of Rational Numbers:

Question:

How many types of rational numbers?

Answer:

There are following types of rational numbers.

Natural Numbers:

All the counting numbers are called **NATURAL NUMBERS**. Its set is represented by N. Therefore, N = {1, 2, 3, 4, 5, 6, 7, 8, 9, ...} is the set of natural numbers. The number of natural number is infinite.

Whole Numbers:

All counting numbers including 0 (zero) form the set of **WHOLE NUMBERS**. Its set is represented by W. Therefore, W = {0, 1, 2, 3, 4, 5, 6...} is the set of the whole numbers. Now, on comparing the set of natural numbers N = {1, 2, 3, 4, 5, 6, 7, 8, 9, ...} is the subset of the whole numbers W = {0, 1, 2, 3, 4, 5, 6...}. Thus, N $\subseteq$ W.

<u>Integers:</u>

All the natural numbers, 0 (zero) and the negative of all the natural numbers from the set of **INTEGERS**. Its set is represented by are denoted by Z or I. Therefore, Z = {-3, -2, -1, 0,1,2,3,...} is the set of integers. Now, we observe that both the set of natural numbers N = {1, 2, 3, 4, 5, 6, 7, 8, 9, ...} and the whole numbers W = {0, 1, 2, 3, 4, 5, 6...} are the subset of integers Z = {-3, -2, -1, 0,1,2,3,...}. Thus, N $\subseteq$ Z and W $\subseteq$ Z $\Rightarrow$ N $\subseteq$ W $\subseteq$ Z.

<u>Rational Numbers:</u>

We know that a number that can be written as $\frac{p}{q}$, where p and q are integers and q $\neq$ 0, is known as **RATIONAL NUMBERS**. Thus, the set of the rational numbers contains all integers and fractions. The set of rational numbers is denoted by Q. Therefore, N $\subseteq$ W $\subseteq$ Z $\subseteq$ Q.

1.3 Decimal Representation of Rational Numbers:

Question:

How do we represent the rational number in decimal form?

Answer:

We know that a number that can be written as $\frac{p}{q}$, where p and q are integers and q $\neq$ 0, is known as rational numbers. Now, to express the rational number $\frac{p}{q}$ in decimal representation, we divide the numerator p by the denominator q. After dividing p by q, we get either the remainder zero or not.

Question:

What are the terminating, non-terminating and recurring decimal?

Answer:

In the division of a rational number $\frac{p}{q}$, when the remainder is zero, then that decimal representation is known as **TERMINATING DECIMAL** and when the remainder is not zero, then that decimal representation is known as **NON-TERMINATING DECIMAL.** Now, in the non-terminating decimal, either the decimal part is repeating, recurring or no repeat, non-recurring. When the decimal part is recurring, then that decimal number is known as **RECURRING DECIMAL** and when the decimal part is non-recurring, then that decimal representation is known as **NON-RECURRING DECIMAL**.

Example: $\frac{1}{8}$ = 0.125, $\frac{1}{2}$ = 0.5, $\frac{1}{25}$ = 0.04 etc are known as **terminating decimal**. In the division of these rational numbers, we get the remainder zero after a finite steps.

Whereas, $\frac{1}{3}$ = 0.33333..., $\frac{9}{11}$ = 0.81818181.... etc are known as recurring decimal. In the division of these rational numbers, we don't get the remainder zero after any finite steps and its decimal part is recurring. The recurring decimal numbers are also represented as

$$\frac{1}{3} = 0.333... = 0.\bar{3} \left(\text{Read as } 0.3 \text{ Bar}\right)$$

$$\frac{9}{11} = 0.818181... = 0.\overline{81} \left(\text{Read as } 0.81 \text{ Bar}\right)$$

Question:

How do we identify that the fraction is terminating or non-terminating decimal numbers without performing devision?

Answer:

In the rational number or fraction $\frac{p}{q}$, if the prime factorization of the denominator q is in the form of $2^m \times 5^n$, it means, the prime factors of the denominator q are 2 or 5 or both. Otherwise, the fraction is non-terminating decimal.

Example:

In $\frac{1}{8}$, we have q =8 = $2^3 \times 5^0$. Thus, $\frac{1}{8}$ is terminating decimal and $\frac{1}{8}$ = 0.125.

In $\frac{1}{25}$, we have q = 25 = $2^0 \times 5^2$. Thus, $\frac{1}{25}$ is terminating decimal and $\frac{1}{25}$ = 0.04.

In $\frac{8}{15}$, we have q = 15 = $3^1 \times 5^1$. Thus, $\frac{8}{15}$ is non-terminating decimal.

Question:

How do we express the recurring decimal as fraction?

Answer:

To express the recurring decimal number in a fraction, we use a formula as below.

$$\text{Recurring Decimal} = \frac{\left(\begin{array}{l}\text{Whole Number Obtained}\\ \text{by Writing Digits in Order}\end{array}\right) - \left(\begin{array}{l}\text{Whole Number Made by}\\ \text{Non-Recurring Digits in Order}\end{array}\right)}{\left(10^{\text{Number of Digits After the Decimal Point}}\right) - \left(10^{\text{Number of Digits After the Decimal Point that do not recur}}\right)}$$

Example: 12.253 = $\frac{12253-122}{990}$ = $\frac{11263}{990}$

1.4 Comparison of Rational Numbers:

Question:

How do we compare two or more rational numbers?

Answer:

To compare two or more rational numbers, we do the following steps:

Step 1: Express each of the rational numbers with the positive denominator.

Step 2: Find the LCM of these positive denominators.

Step 3: Express each of the rational number with this LCM as the common denominator.

Step 4: The numerator having the greater numerator is greater.

Question:

Arrange the numbers $\frac{-3}{5}$, $\frac{7}{-10}$ and $\frac{-5}{8}$.

Answer:

We are given $\frac{-3}{5}$, $\frac{7}{-10} = \frac{-7}{10}$ and $\frac{-5}{8}$. LCM of (5, 10 and 8) = 40

$\frac{-3}{5} = \frac{-3\times8}{5\times8} = -\frac{24}{40}$

$\frac{-7}{10} = \frac{-7\times4}{10\times4} = -\frac{28}{40}$

$\frac{-5}{8}=\frac{-5\times5}{8\times5}=-\frac{25}{40}$

Thus, we get $\frac{-28}{40}<\frac{-25}{40}<\frac{-24}{40}\Rightarrow\frac{-7}{10}<\frac{-5}{8}<\frac{-3}{5}$.

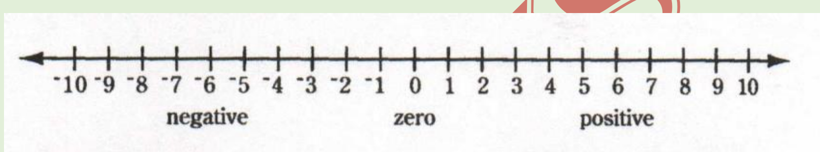
1.5 Representation of Rational Numbers on Number Lines:

Question:

How do we represent the rational numbers on number line?

Answer:

To represent a rational number, we draw a line. Take a point O on it. Mark it as zero (0). Set off equal distances on the right of O and the left of O. These distances is known as a unit length. Clearly, the points A, B, C, D, E represent the integers 1, 2, 3, 4 and 5 respectively and the points A', B' C', D' and E' represent the integers -1, -2, -3, -4 and -5 respectively.

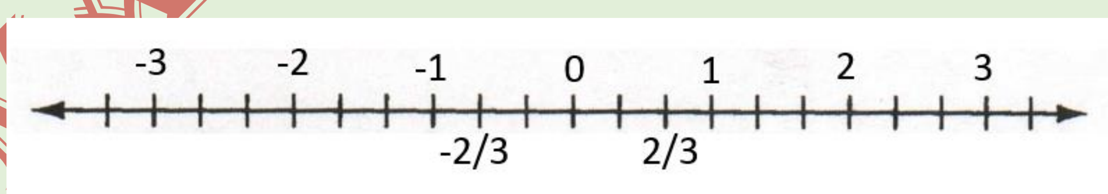


Thus, we represent any integer by a point on the number line. Clearly, every positive integer lies to the right of O and every negative integers lies to the left of O. Similarly, we represent other rational numbers.

Question:

Represent $\frac{2}{3}$ and $-\frac{2}{3}$ on number line.

Answer:



1.6 Addition of Rational Numbers

Question:

How to add two rational numbers if the given rational numbers have same denominator?

Answer:

When the given rational numbers have same denominator, then we use the method as $\frac{a}{b}+\frac{c}{b}=\frac{a+c}{b}$.

Question:

Add the rational numbers, $\frac{-7}{9}+\frac{11}{9}$.

Answer:

$\frac{-7}{9}+\frac{11}{9}=\frac{-7+11}{9}=\frac{4}{9}$

Question:

How to add two rational numbers if the given rational numbers have different denominator?

Answer:

When the given rational numbers have same denominator, then we take the LCM of their denominators and express each of the given numbers with this LCM as the common denominator. Now, we add these numbers as $\frac{a}{b}+\frac{c}{b}=\frac{a+c}{b}$.

Question:

Find the sum $\frac{-5}{6}+\frac{4}{9}$.

Answer:

The denominator of the given rational numbers are 6 and 9. LCM of (6, 9) = 18. Thus, $\frac{-5}{6}=\frac{-5\times3}{6\times3}=\frac{-15}{18}$ and $\frac{4}{9}=\frac{4\times2}{9\times2}=\frac{8}{18}$.

Now, $\frac{-5}{6}+\frac{4}{9}=\frac{-15}{18}+\frac{8}{18}=\frac{-15+8}{18}=\frac{-7}{18}$.

1.7 Properties of Addition of Rational Numbers

Question:

What are the properties of addition of rational numbers?

Answer:

There are following properties of addition of rational numbers:

Property 1: The sum of two rational numbers is always a rational number. It means, if $\frac{a}{b}$ and $\frac{c}{d}$ are rational numbers, then $\left(\frac{a}{b}+\frac{c}{d}\right)$ are rational number. This property is known as **CLOSURE PROPERTY**.

Property 2: Two rational numbers can be added in any order. It means, if $\frac{a}{b}$ and $\frac{c}{d}$ are rational numbers, then $\left(\frac{a}{b}+\frac{c}{d}\right)$ = $\left(\frac{c}{d}+\frac{a}{b}\right)$. This property is known as **COMMUTATIVE PROPERTY**.

Property 3: While adding three rational numbers, they can be grouped in any order. It means, if $\frac{a}{b}$, $\frac{c}{d}$ and $\frac{e}{f}$ are three rational numbers, then ($\frac{a}{b}$ + $\frac{c}{d}$) + $\frac{e}{f}$ = $\frac{a}{b}$ + ($\frac{c}{d}$ + $\frac{e}{f}$) . This property is known as **ASSOCIATIVE PROPERTY**.

Property 4: 0 (zero) is a rational number such that the sum of any rational number and 0 (zero) is the rational number itself. Thus, ($\frac{a}{b}$ + 0) =(0 + $\frac{a}{b}$)= $\frac{a}{b}$. This property is known as **EXISTENCE OF ADDITIVE IDENTITY** and 0 (zero) is known as **ADDITIVE IDENTITY** for rational numbers.

Property 5: For any rational number $\frac{a}{b}$, there exist a rational number $\frac{-a}{b}$ such that ($\frac{a}{b}$ + $\frac{-a}{b}$) = 0. This property is known as **EXISTENCE OF ADDITIVE INVERSE** and $\frac{-a}{b}$ is known as **ADDITIVE INVERSE** of $\frac{a}{b}$.

1.8 Subtraction of Rational Numbers

Question:

How to subtract two rational numbers?

Answer:

For rational number $\frac{a}{b}$ and $\frac{c}{d}$, we subtract as ($\frac{a}{b}$ - $\frac{c}{d}$) = ($\frac{a}{b}$ + $\frac{-c}{d}$) = ($\frac{a}{b}$ + Additive Inverse of $\frac{c}{d}$).

Question:

Subtract $\frac{3}{4}$ from $\frac{2}{3}$

Answer:

$\frac{2}{3} - \frac{3}{4} = \frac{2}{3} + \frac{-3}{4} = \frac{8+(-9)}{12} = \frac{-1}{12}$.

1.9 Properties of Subtraction of Rational Numbers

Question:

What are the properties of subtraction of rational numbers?

Answer:

There are following properties of subtraction of rational numbers:

Property 1: The subtraction of two rational numbers is always a rational number. It means, if $\frac{a}{b}$ and $\frac{c}{d}$ are rational numbers, then $(\frac{a}{b} - \frac{c}{d})$ is rational number. This property is known as **CLOSURE PROPERTY**.

Property 2: Two rational numbers **cannot** be subtracted in any order. It means, if $\frac{a}{b}$ and $\frac{c}{d}$ are rational numbers, then $(\frac{a}{b} - \frac{c}{d}) \neq (\frac{c}{d} - \frac{a}{b})$. It means, the subtraction **does not** follow the rule **COMMUTATIVE PROPERTY** as in addition. Similarly, the subtraction does not follow **ASSOCIATIVE PROPERTY**.

1.10 Multiplication of Rational Numbers

Question:

How do we multiply two rational numbers?

Answer:

For two rational numbers $\frac{a}{b}$ and $\frac{c}{d}$, we get the multiplication as $\frac{a}{b} \times \frac{c}{d} = \frac{a \times c}{b \times d}$.

Question:

Multiply $\frac{-3}{7} \times \frac{14}{5}$

Answer:

We multiply as $\frac{-3}{7} \times \frac{14}{5} = \frac{-6}{5}$.

1.11 Properties of Multiplication of Rational Numbers

Question:

What are the properties of multiplication of rational numbers?

Answer:

There are following properties of multiplication of rational numbers:

Property 1: The multiplication or the product of two rational numbers is always a rational number. It means, if $\frac{a}{b}$ and $\frac{c}{d}$ are rational numbers, then $\frac{a}{b} \times \frac{c}{d}$ is rational number. This property is known as **CLOSURE PROPERTY**.

Property 2: Two rational numbers can be multiplied in any order. It means, if $\frac{a}{b}$ and $\frac{c}{d}$ are rational numbers, then $\frac{a}{b} \times \frac{c}{d} = \frac{c}{d} \times \frac{a}{b}$. This property is known as **COMMUTATIVE PROPERTY**.

Property 3: While multiplying three rational numbers, they can be grouped in any order. It means, if $\frac{a}{b}$, $\frac{c}{d}$ and $\frac{e}{f}$ are three rational numbers, then $(\frac{a}{b} \times \frac{c}{d}) \times \frac{e}{f} = \frac{a}{b} \times (\frac{c}{d} \times \frac{e}{f})$. This property is known as **ASSOCIATIVE PROPERTY**.

Property 4: 1 (one) is a rational number such that the product of any rational number and 1 is the rational number itself. Thus, $(\frac{a}{b} \times 1) = (1 \times \frac{a}{b}) = \frac{a}{b}$. This property is known as **EXISTENCE OF MULTIPLICATIVE IDENTITY** and 1 is known as **MULTIPLICATIVE IDENTITY** for rational numbers.

Property 5: For any rational number $\frac{a}{b}$, there exist a rational number $\frac{b}{a}$ such that $(\frac{a}{b} \times \frac{b}{a}) = 1$. This property is known as **EXISTENCE OF MULTIPLICATIVE INVERSE** and $\frac{b}{a}$ is known as **MULTIPLICATIVE INVERSE or RECIPROCAL** of $\frac{a}{b}$. Note that 0 (zero) has no reciprocal, because $\frac{1}{0}$ is not defined. The reciprocal of 1 is 1 and the reciprocal of (-1) is (-1).

Property 6: For any three rational numbers $\frac{a}{b}$, $\frac{c}{d}$ and $\frac{e}{f}$, we have $\frac{a}{b} \times (\frac{c}{d} + \frac{e}{f}) = (\frac{a}{b} \times \frac{c}{d}) + (\frac{a}{b} \times \frac{e}{f})$. This property is known as **DISTRIBUTIVE LAW OF MULTIPLICATION OVER ADDITION PROPERTY**.

Property 7: If we multiply any rational number with 0 (zero), then the result is always 0 (zero). It means, $(\frac{a}{b} \times 0) = (0 \times \frac{a}{b}) = 0$. This property is known as **MULTIPLICATIVE PROPERTY OF 0 (ZERO).**

1.12 Division of Rational Numbers

Question:

How do we divide two rational numbers?

Answer:

If $\frac{a}{b}$ and $\frac{c}{d}$ are two rational numbers such that $\frac{c}{d} \neq 0$, then the division of $\frac{a}{b}$ by $\frac{c}{d}$ is defined as ($\frac{a}{b} \div \frac{c}{d}$) = ($\frac{a}{b} \times \frac{d}{c}$). It means, when $\frac{a}{b}$ is divided by $\frac{c}{d}$, then $\frac{a}{b}$ is known as dividend and $\frac{c}{d}$ is known as divisor and the product/result is known as the quotient of the division.

Question:

Divide $\frac{9}{16}$ by $\frac{5}{8}$.

Answer:

$\frac{9}{16} \div \frac{5}{8} = \frac{9}{16} \times \frac{8}{5} = \frac{9}{10}$.

1.13 Properties of Division of Rational Numbers

Question:

What are the properties of division of rational numbers?

Answer:

There are following properties of division of rational numbers:

Property 1: If $\frac{a}{b}$ and $\frac{c}{d}$ are two rational numbers such that $\frac{c}{d} \neq 0$, then ($\frac{a}{b} \div \frac{c}{d}$) is also a rational number. This property is known as **CLOSURE PROPERTY**.

Property 2: For every rational number, we have then ($\frac{a}{b} \div 1$) = $\frac{a}{b}$. This property is known as **PROPERTY OF 1**.

Property 3: For every rational number, we have then ($\frac{a}{b} \div \frac{b}{a}$) = 1. Here, $\frac{a}{b}$ and $\frac{b}{a}$ are **RECIPROCAL TO EACH OTHER**.

1.14 Finding one or more than one rational number between two Rational Numbers

Question:

How do we find a rational number between two rational number?

Answer:

If x and y are two rational numbers such that x < y, then $\frac{x+y}{2}$ is the rational number between x and y.

Question:

Find a rational number between $\frac{1}{3}$ and $\frac{1}{2}$.

Answer:

Required Number = $(\frac{1}{3}+\frac{1}{2})/2=(\frac{5}{6})/2=\frac{5}{12}$.

Question:

Find three rational number between 3 and 5.

Answer:

First number between 3 and 5 = $\frac{3+5}{2}=4$

Second number between 3 and 4 = $\frac{3+4}{2}=\frac{7}{2}$

Third number between 4 and 5 = $\frac{4+5}{2}=\frac{9}{2}$

Thus, the three rational number between 3 and 5 are 4, 7/2, 9/2.

Question:

Find 9 rational number between 1 and 2.

Answer:

Since 1 = $\frac{10}{10}$ and 2 = $\frac{20}{10}$, the nine rational numbers between 1 and 2 are

$\frac{11}{10},\frac{12}{10},\frac{13}{10},\frac{14}{10},\frac{15}{10},\frac{16}{10},\frac{17}{10},\frac{18}{10},\frac{19}{10}$